# 『线性最优滤波小结』

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Spring 2024

# 0. Outline

卡尔曼滤波算法可以从不同的角度审视和推导，通常可以更加方便地建立滤波公式，同时也有利于快速记忆。首先，对于一步预测，即时间修正，不涉及新的量测信息，相对简单。

这里不再复述标准假设。设已知 $(\hat{\boldsymbol{x}}_k, \boldsymbol{P}_k)$，由系统的状态方程

$$\boldsymbol{x}_{k+1} = \boldsymbol{\Phi}_{k+1,k}\boldsymbol{x}_k + \boldsymbol{\Psi}_{k+1,k}\boldsymbol{u}_k + \boldsymbol{\Gamma}_k\boldsymbol{w}_k \tag{1}$$

可知

$$\boxed{\begin{aligned}\hat{\boldsymbol{x}}_{k+1|k} &= E[\boldsymbol{x}_{k+1}|\boldsymbol{Y}_1^k] = \boldsymbol{\Phi}_{k+1,k}E[\boldsymbol{x}_k|\boldsymbol{Y}_1^k] + \boldsymbol{\Psi}_{k+1,k}\boldsymbol{u}_k \\ &= \boldsymbol{\Phi}_{k+1,k}\hat{\boldsymbol{x}}_k + \boldsymbol{\Psi}_{k+1,k}\boldsymbol{u}_k\end{aligned}} \tag{2}$$

由 (1) 和 (2) 式可导出

$$\boxed{\boldsymbol{P}_{k+1|k} = \boldsymbol{\Phi}_{k+1,k}\boldsymbol{P}_k\boldsymbol{\Phi}_{k+1,k}^T + \boldsymbol{\Gamma}_k\boldsymbol{Q}_k\boldsymbol{\Gamma}_k^T} \tag{3}$$

(2) 和 (3) 便构成了最优一步预测公式（时间修正）。

# 1. 融合估计与 KF

当获得 $k+1$ 次量测，即

$$\boldsymbol{y}_{k+1} = \boldsymbol{H}_{k+1}\boldsymbol{x}_{k+1} + \boldsymbol{v}_{k+1}, \quad \boldsymbol{v}_{k+1} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{R}_{k+1}) \tag{4}$$

可以建立此时的一个最优估计 (最小二乘)

$$\begin{cases} \hat{\boldsymbol{x}}^{a}_{k+1} = (\boldsymbol{H}^{T}_{k+1}\boldsymbol{R}^{-1}_{k+1}\boldsymbol{H}_{k+1})^{-1}\boldsymbol{H}^{T}_{k+1}\boldsymbol{R}^{-1}_{k+1}\boldsymbol{y}_{k+1} \\ \boldsymbol{P}^{a}_{k+1} = (\boldsymbol{H}^{T}_{k+1}\boldsymbol{R}^{-1}_{k+1}\boldsymbol{H}_{k+1})^{-1} \end{cases} \tag{5}$$

将 $(\hat{\boldsymbol{x}}^a_{k+1}, \boldsymbol{P}^a_{k+1})$ 与 $(\hat{\boldsymbol{x}}_{k+1|k}, \boldsymbol{P}_{k+1|k})$ 进行融合，则得 $k+1$ 时刻状态的最优估计如下：

$$\begin{cases} \hat{\boldsymbol{x}}_{k+1} = \boldsymbol{P}_{k+1}(\boldsymbol{P}^{-1}_{k+1|k}\hat{\boldsymbol{x}}_{k+1|k} + \boldsymbol{H}^T_{k+1}\boldsymbol{R}^{-1}_{k+1}\boldsymbol{y}_{k+1}) \\ \boldsymbol{P}_{k+1} = (\boldsymbol{P}^{-1}_{k+1|k} + \boldsymbol{H}^T_{k+1}\boldsymbol{R}^{-1}_{k+1}\boldsymbol{H}_{k+1})^{-1} \end{cases} \tag{6}$$

上式可以变换为其他等价形式。

考虑到卡尔曼滤波的结构为

$$\begin{aligned} \hat{\boldsymbol{x}}_{k+1} &= \hat{\boldsymbol{x}}_{k+1|k} + \boldsymbol{K}_{k+1}(\boldsymbol{y}_{k+1} - \boldsymbol{H}_{k+1}\hat{\boldsymbol{x}}_{k+1|k}) \\ &= \underline{(\boldsymbol{I} - \boldsymbol{K}_{k+1}\boldsymbol{H}_{k+1})\hat{\boldsymbol{x}}_{k+1|k} + \boldsymbol{K}_{k+1}\boldsymbol{y}_{k+1}} \end{aligned} \tag{7}$$

比较 (6) 与 (7)，可知

$$\boxed{\begin{aligned} \boldsymbol{K}_{k+1} &= \boldsymbol{P}_{k+1}\boldsymbol{H}_{k+1}^{T}\boldsymbol{R}_{k+1}^{-1} \quad (8) \\ \boldsymbol{P}_{k+1} &= (\boldsymbol{I}-\boldsymbol{K}_{k+1}\boldsymbol{H}_{k+1})\boldsymbol{P}_{k+1|k} \quad (9) \end{aligned}}$$

(7)、(8)、(9) 是常见的卡尔曼滤波公式。

## 2. 最小二乘与 KF

在 $k+1$ 时刻，我们有

$$\underline{\hat{\boldsymbol{x}}_{k+1|k} = \boldsymbol{x}_{k+1} + \boldsymbol{\epsilon}_{k+1}}, \boldsymbol{\epsilon}_{k+1} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{P}_{k+1|k}) \quad (10)$$

$$\boldsymbol{y}_{k+1} = \boldsymbol{H}_{k+1}\boldsymbol{x}_{k+1} + \boldsymbol{v}_{k+1}, \quad \boldsymbol{v}_{k+1} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{R}_{k+1}) \quad (11)$$

将 (10) 也视为量测方程之一，于是

$$\mathcal{H} = \begin{bmatrix} \boldsymbol{I}_{n\times n} \\ \boldsymbol{H}_{k+1} \end{bmatrix}, \quad \mathcal{V} = \begin{bmatrix} \boldsymbol{\epsilon}_{k+1} \\ \boldsymbol{v}_{k+1} \end{bmatrix}$$

$$E\mathcal{V}\mathcal{V}^T = \begin{bmatrix} \boldsymbol{P}_{k+1|k} & 0 \\ 0 & \boldsymbol{R}_{k+1} \end{bmatrix} \overset{\text{def}}{=} \mathcal{W}^{-1}, \Rightarrow \mathcal{W} = \begin{bmatrix} \boldsymbol{P}_{k+1|k}^{-1} & 0 \\ 0 & \boldsymbol{R}_{k+1}^{-1} \end{bmatrix}$$

由加权最小二乘估计可得

$$\mathcal{H}^T\mathcal{W} = \begin{bmatrix} \boldsymbol{I}_{n\times n} & \boldsymbol{H}_{k+1}^T \end{bmatrix} \begin{bmatrix} \boldsymbol{P}_{k+1|k}^{-1} & 0 \\ 0 & \boldsymbol{R}_{k+1}^{-1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}_{k+1|k}^{-1} & \boldsymbol{H}_{k+1}\boldsymbol{R}_{k+1}^{-1} \end{bmatrix}$$

$$\mathcal{H}^T\mathcal{W}\mathcal{H} = \begin{bmatrix} \boldsymbol{P}_{k+1|k}^{-1} & \boldsymbol{H}_{k+1}^T\boldsymbol{R}_{k+1}^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_{n\times n} \\ \boldsymbol{H}_{k+1} \end{bmatrix} = \boldsymbol{P}_{k+1|k}^{-1} + \boldsymbol{H}_{k+1}^T\boldsymbol{R}_{k+1}^{-1}\boldsymbol{H}_{k+1}$$

其中

$$\mathcal{H}^T\mathcal{W} = \begin{bmatrix} \boldsymbol{I}_{n\times n} & \boldsymbol{H}_{k+1}^T \end{bmatrix} \begin{bmatrix} \boldsymbol{P}_{k+1|k}^{-1} & 0 \\ 0 & \boldsymbol{R}_{k+1}^{-1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}_{k+1|k}^{-1} & \boldsymbol{H}_{k+1}\boldsymbol{R}_{k+1}^{-1} \end{bmatrix}$$

$$\mathcal{H}^T\mathcal{W}\mathcal{H} = \begin{bmatrix} \boldsymbol{P}_{k+1|k}^{-1} & \boldsymbol{H}_{k+1}^T\boldsymbol{R}_{k+1}^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_{n\times n} \\ \boldsymbol{H}_{k+1} \end{bmatrix} = \boldsymbol{P}_{k+1|k}^{-1} + \boldsymbol{H}_{k+1}^T\boldsymbol{R}_{k+1}^{-1}\boldsymbol{H}_{k+1}$$

因此

$$\hat{\boldsymbol{x}}_{k+1} = \boldsymbol{P}_{k+1}\left(\boldsymbol{P}_{k+1|k}^{-1}\hat{\boldsymbol{x}}_{k+1|k} + \boldsymbol{H}_{k+1}\boldsymbol{R}_{k+1}^{-1}\boldsymbol{y}_{k+1}\right) \tag{12}$$

其他讨论同上小节，不再赘述。

# 3. 极大验后估计与 KF

极大验后估计指

$$f(\boldsymbol{x}|\boldsymbol{y}) = \frac{f(\boldsymbol{y}|\boldsymbol{x})f(\boldsymbol{x})}{f(\boldsymbol{y})} \Rightarrow \max$$

对于我们目前的问题，对应的概率密度函数为

$$f(\boldsymbol{y}|\boldsymbol{x}) = f(\boldsymbol{y}_{k+1}|\boldsymbol{x}_{k+1}) = \mathcal{N}(\boldsymbol{H}_{k+1}\boldsymbol{x}_{k+1}, \boldsymbol{R}_{k+1})$$
$$f(\boldsymbol{x}) = f(\boldsymbol{x}_{k+1}) = \mathcal{N}(\hat{\boldsymbol{x}}_{k+1|k}, \boldsymbol{P}_{k+1|k})$$

极大验后概率密度相当于

$$
\begin{aligned}
J(\boldsymbol{x}_{k+1}) = &\frac{1}{2}(\boldsymbol{y}_{k+1} - \boldsymbol{H}_{k+1}\boldsymbol{x}_{k+1})^T \boldsymbol{R}_{k+1}^{-1}(\boldsymbol{y}_{k+1} - \boldsymbol{H}_{k+1}\boldsymbol{x}_{k+1}) \\
&+ \frac{1}{2}(\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1|k})^T \boldsymbol{P}_{k+1|k}^{-1}(\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1|k}) \Rightarrow \min
\end{aligned}
$$

所以

$$
\boldsymbol{P}_{k+1|k}^{-1}\left(\hat{\boldsymbol{x}}_{k+1} - \hat{\boldsymbol{x}}_{k+1|k}\right) - \boldsymbol{H}_{k+1}^T \boldsymbol{R}_{k+1}^{-1}\left(\boldsymbol{y}_{k+1} - \boldsymbol{H}_{k+1}\hat{\boldsymbol{x}}_{k+1}\right) = 0
$$

$$
\Rightarrow \boxed{\begin{cases} \hat{\boldsymbol{x}}_{k+1} = \boldsymbol{P}_{k+1}\left(\boldsymbol{P}_{k+1|k}^{-1}\hat{\boldsymbol{x}}_{k+1|k} + \boldsymbol{H}_{k+1}^T \boldsymbol{R}_{k+1}^{-1}\boldsymbol{y}_{k+1}\right) \\ \boldsymbol{P}_{k+1} = \left(\boldsymbol{P}_{k+1|k}^{-1} + \boldsymbol{H}_{k+1}^T \boldsymbol{R}_{k+1}^{-1}\boldsymbol{H}_{k+1}\right)^{-1} \end{cases}}
$$

# 4. 最小方差估计与 KF

已知

$$
\begin{aligned}
\boldsymbol{y}_{k+1} &= \boldsymbol{H}_{k+1}\boldsymbol{x}_{k+1} + \boldsymbol{v}_{k+1} \\
\boldsymbol{x}_{k+1} &\sim \mathcal{N}(\hat{\boldsymbol{x}}_{k+1|k}, \boldsymbol{P}_{k+1|k}) \\
\boldsymbol{v}_{k+1} &\sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{R}_{k+1})
\end{aligned}
$$

由（线性）最小方差估计理论可知

$$
\hat{\boldsymbol{x}}_{k+1} = \hat{\boldsymbol{x}}_{k+1|k} + \boldsymbol{K}_{k+1}(\boldsymbol{y}_{k+1} - \boldsymbol{H}_{k+1}\hat{\boldsymbol{x}}_{k+1|k}) \tag{13}
$$

其中

$$
\boldsymbol{K}_{k+1} = \boldsymbol{P}_{\boldsymbol{x}_{k+1}\boldsymbol{y}_{k+1}}\boldsymbol{P}_{\boldsymbol{y}_{k+1}}^{-1}
$$

容易导出

$$\boxed{\begin{aligned} \boldsymbol{P}_{\boldsymbol{y}_{k+1}} &= \boldsymbol{H}_{k+1}\boldsymbol{P}_{k+1|k}\boldsymbol{H}_{k+1}^T + \boldsymbol{R}_{k+1} \\ \boldsymbol{P}_{\boldsymbol{x}_{k+1}\boldsymbol{y}_{k+1}} &= \boldsymbol{P}_{k+1|k}\boldsymbol{H}_{k+1}^T \end{aligned}}$$

所以 Kalman 增益为

$$\underline{\boldsymbol{K}_{k+1} = \boldsymbol{P}_{k+1|k}\boldsymbol{H}_{k+1}^T(\boldsymbol{H}_{k+1}\boldsymbol{P}_{k+1|k}\boldsymbol{H}_{k+1}^T + \boldsymbol{R}_{k+1})^{-1}} \tag{14}$$

另外，估计误差协方差为 $(\underline{\boldsymbol{P}_{\hat{\boldsymbol{x}}} = \boldsymbol{P}_{\boldsymbol{x}} - \boldsymbol{P}_{\boldsymbol{xy}}\boldsymbol{P}_{\boldsymbol{y}}^{-1}\boldsymbol{P}_{\boldsymbol{xy}}^T = \boldsymbol{P}_{\tilde{\boldsymbol{x}}_{MV}}})$

$$\begin{aligned} \boldsymbol{P}_{k+1} &= \boldsymbol{P}_{k+1|k} - \boldsymbol{P}_{k+1|k}\boldsymbol{H}_{k+1}^T(\boldsymbol{H}_{k+1}\boldsymbol{P}_{k+1|k}\boldsymbol{H}_{k+1}^T + \boldsymbol{R}_{k+1})^{-1}\boldsymbol{H}_{k+1}\boldsymbol{P}_{k+1|k} \\ &= \underline{(\boldsymbol{I} - \boldsymbol{K}_{k+1}\boldsymbol{H}_{k+1})\boldsymbol{P}_{k+1|k}} \end{aligned} \tag{15}$$

另外，由 (13) 可知

$$
\begin{aligned}
\tilde{\boldsymbol{x}}_{k+1} = \boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1} &= \tilde{\boldsymbol{x}}_{k+1|k} - \boldsymbol{K}_{k+1}\left(\boldsymbol{y}_{k+1} - \boldsymbol{H}_{k+1}\hat{\boldsymbol{x}}_{k+1|k}\right) \\
&= \tilde{\boldsymbol{x}}_{k+1|k} - \boldsymbol{K}_{k+1}\left(\boldsymbol{H}_{k+1}\tilde{\boldsymbol{x}}_{k+1|k} + \boldsymbol{v}_{k+1}\right) \\
&= (\boldsymbol{I} - \boldsymbol{K}_{k+1}\boldsymbol{H}_{k+1})\tilde{\boldsymbol{x}}_{k+1|k} - \boldsymbol{K}_{k+1}\boldsymbol{v}_{k+1}
\end{aligned}
$$

于是

$$
\underline{\boldsymbol{P}_{k+1} = (\boldsymbol{I} - \boldsymbol{K}_{k+1}\boldsymbol{H}_{k+1})\boldsymbol{P}_{k+1|k}(\boldsymbol{I} - \boldsymbol{K}_{k+1}\boldsymbol{H}_{k+1})^T + \boldsymbol{K}_{k+1}\boldsymbol{R}_{k+1}\boldsymbol{K}_{k+1}^T} \tag{16}
$$

这是一个有利于保证正定对称性的计算公式，而且对于非最优的 $\boldsymbol{K}_{k+1}$ 也成立!

# 5. 相关噪声的滤波算法

这里从最小方差估计基本原理出发，给出不同于 YC Ho 方法的结果。首先，回顾过程噪声与量测噪声相关的滤波问题。

系统状态方程和量测方程为

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{\Phi}_{k+1,k}\boldsymbol{x}_k + \boldsymbol{\Psi}_{k+1,k}\boldsymbol{u}_k + \boldsymbol{\Gamma}_k\boldsymbol{w}_k \\ \boldsymbol{y}_{k+1} &= \boldsymbol{H}_{k+1}\boldsymbol{x}_{k+1} + \boldsymbol{v}_{k+1} \end{aligned}$$

过程噪声及量测噪声特性为

$$
\begin{aligned}
\boldsymbol{w}_k &\sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{Q}_k) \\
\boldsymbol{v}_k &\sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{R}_k) \\
E\boldsymbol{w}_k\boldsymbol{w}_j^T &= \boldsymbol{Q}_k\delta_{kj} \\
E\boldsymbol{v}_k\boldsymbol{v}_j^T &= \boldsymbol{R}_k\delta_k j \\
\underline{E\boldsymbol{w}_k\boldsymbol{v}_j^T} &\underline{= \boldsymbol{C}_k\delta_{k,j-1} \quad (E\boldsymbol{w}_k\boldsymbol{v}_{k+1}^T = \boldsymbol{C}_k)}
\end{aligned}
$$

注意这里关于两个噪声序列相关的假设和 YC Ho 描述的有一点差异。假设已经获得 $k$ 时刻的无偏最优估计 $(\hat{\boldsymbol{x}}_k, \boldsymbol{P}_k)$, 即

$$
\boldsymbol{x}_k \sim \mathcal{N}(\hat{\boldsymbol{x}}_k, \boldsymbol{P}_k)
$$

可以认为 $\boldsymbol{x}_k$ 与 $\boldsymbol{w}_k$ 和 $\boldsymbol{v}_{k+1}$ 无关，根据状态方程可得

$$\begin{aligned}\hat{\boldsymbol{x}}_{k+1|k} &= \boldsymbol{\Phi}_{k+1,k}\hat{\boldsymbol{x}}_k + \boldsymbol{\Psi}_{k+1,k}\boldsymbol{u}_k \\ \boldsymbol{P}_{k+1|k} &= \boldsymbol{\Phi}_{k+1,k}\boldsymbol{P}_k\boldsymbol{\Phi}_{k+1,k}^T\end{aligned}$$

现在的问题化为

$$\begin{aligned}\boldsymbol{y}_{k+1} &= \boldsymbol{H}_{k+1}\boldsymbol{x}_{k+1} + \boldsymbol{v}_{k+1} \\ \boldsymbol{x}_{k+1} &\sim \mathcal{N}(\hat{\boldsymbol{x}}_{k+1|k}, \boldsymbol{P}_{k+1|k})\end{aligned}$$

注意，此时 $\boldsymbol{x}_{k+1}$ 与 $\boldsymbol{v}_{k+1}$ 相关。由（线性）最小方差估计理论，可知

$$\hat{\boldsymbol{x}}_{k+1} = \hat{\boldsymbol{x}}_{k+1|k} + \boldsymbol{K}_{k+1}(\boldsymbol{y}_{k+1} - \boldsymbol{H}_{k+1}\hat{\boldsymbol{x}}_{k+1|k})$$

由于

$$
\begin{aligned}
\boldsymbol{P}_{\boldsymbol{x}_{k+1}\boldsymbol{y}_{k+1}} &= E[(\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1|k})(\boldsymbol{H}_{k+1}\boldsymbol{x}_{k+1} + \boldsymbol{v}_{k+1} - \boldsymbol{H}_{k+1}\hat{\boldsymbol{x}}_{k+1|k})^T] \\
&= E[[\mathring{\boldsymbol{x}}_{k+1|k}(\boldsymbol{H}_{k+1}[\mathring{\boldsymbol{x}}_{k+1|k} + \boldsymbol{v}_{k+1})^T] \\
&= \boldsymbol{P}_{k+1|k}\boldsymbol{H}_{k+1}^T \underline{+ \boldsymbol{\Gamma}_k\boldsymbol{C}_k} \\
\boldsymbol{P}_{\boldsymbol{y}_{k+1}} &= E[\mathring{\boldsymbol{y}}_{k+1}[\mathring{\boldsymbol{y}}_{k+1}^T \\
&= E[(\boldsymbol{H}_{k+1}[\mathring{\boldsymbol{x}}_{k+1|k} + \boldsymbol{v}_{k+1})(\boldsymbol{H}_{k+1}[\mathring{\boldsymbol{x}}_{k+1|k} + \boldsymbol{v}_{k+1})^T] \\
&= \boldsymbol{H}_{k+1}\boldsymbol{P}_{k+1|k}\boldsymbol{H}_{k+1}^T \underline{+ \boldsymbol{H}_{k+1}\boldsymbol{\Gamma}_k\boldsymbol{C}_k + \boldsymbol{C}_k^T\boldsymbol{\Gamma}_k^T\boldsymbol{H}_{k+1}^T} + \boldsymbol{R}_{k+1}
\end{aligned}
$$

因此

$$
\begin{aligned}
\boldsymbol{K}_{k+1} &= \boldsymbol{P}_{\boldsymbol{x}_{k+1}\boldsymbol{y}_{k+1}}\boldsymbol{P}_{\boldsymbol{y}_{k+1}}^{-1} \\
&= (\boldsymbol{P}_{k+1|k}\boldsymbol{H}_{k+1}^T + \boldsymbol{\Gamma}_k\boldsymbol{C}_k) \\
&\times (\boldsymbol{H}_{k+1}\boldsymbol{P}_{k+1|k}\boldsymbol{H}_{k+1}^T + \boldsymbol{H}_{k+1}\boldsymbol{\Gamma}_k\boldsymbol{C}_k + \boldsymbol{C}_k^T\boldsymbol{\Gamma}_k^T\boldsymbol{H}_{k+1}^T + \boldsymbol{R}_{k+1})^{-1}
\end{aligned}
$$

关于滤波的协方差，有

$$
\begin{aligned}
\boldsymbol{P}_{k+1} &= E[\mathring{\boldsymbol{x}}_{k+1}[\mathring{\boldsymbol{x}}_{k+1}^T = \boldsymbol{P}_{\boldsymbol{x}_{k+1}} - \boldsymbol{P}_{\boldsymbol{x}_{k+1}\boldsymbol{y}_{k+1}}\boldsymbol{P}_{\boldsymbol{y}_{k+1}}^{-1}\boldsymbol{P}_{\boldsymbol{x}_{k+1}\boldsymbol{y}_{k+1}}^T \\
&= \boldsymbol{P}_{k+1|k} - \boldsymbol{K}_{k+1}(\boldsymbol{P}_{k+1|k}\boldsymbol{H}_{k+1}^T + \boldsymbol{\Gamma}_k\boldsymbol{C}_k)^T \\
&= (\boldsymbol{I} - \boldsymbol{K}_{k+1}\boldsymbol{H}_{k+1})\boldsymbol{P}_{k+1|k} - \underline{\boldsymbol{K}_{k+1}\boldsymbol{C}_k^T\boldsymbol{\Gamma}_k^T}
\end{aligned}
$$

**Example 5.1** 考虑如下标量系统:

$$
\begin{aligned}
x_{k+1} &= 0.8x_k + w_k \\
y_{k+1} &= x_{k+1} + v_{k+1} \\
Ew_kw_j^T &= 1.0\delta_{kj} \\
Ev_kv_j^T &= 0.1\delta_{kj} \\
Ew_kv_j^T &= C\delta_{k,j-1} \quad (Ew_kv_{k+1}^T = C)
\end{aligned}
$$

我们用标准 *Kalman* 滤波算法（不考虑过程噪声和量测噪声的相关性）和考虑过程噪声和量测噪声相关性的滤波算法进行仿真，仿真 *50* 步后状态估计误差的方差比较见表 *1*.

Table 1: 噪声与量测噪声相关仿真

| 噪声相关系数 $C$ | 标准滤波算法 ($C=0$) | 修正滤波算法 (采用 $C$ 值进行修正) |
|---|---|---|
| 0 | 0.076 | 0.076 |
| 0.25 | 0.030 | 0.019 |
| -0.25 | 0.117 | 0.052 |

□

# Questions?